

# Summary of Major Concepts, Principles, and Functions of Logic

Revision January 3, 2008

## The Three Laws of Thought . . . Are Equally Laws of Things

**Identity:** A is A.

**Contradiction:** A is not non-A.

**Excluded middle:** A is either B or non-B.

**(God's Truth Denotes God's Reality)**

(A thing is what it is.)

(No thing simultaneously has a character and has it not.)

(No thing is both what it is and not what it is.)

## Definition

"The **definition** of anything is the statement of its essence: what makes it that, and not something else." (Joseph, 72) **Six Rules of Definition:** A definition must (1) "give the essence of that which is to be defined"; (2) "be *per genus et differentiam*" (of kind and varieties); (3) "be commensurate with that which is to be defined, i.e. be applicable to everything included in the species defined, and to nothing else"; (4) "not, directly or indirectly, define the subject by itself"; (5) "not be in negative where it can be in positive terms"; (6) "not be expressed in obscure or figurative language." (Joseph, 111-115)

## Parts of a Categorical Proposition

Terms: subject and predicate

Copula—linking verb

Quantifier(s): *all, some, most, few*, etc.; modify terms

Qualifier: silent affirmative, *no* or *not* negative; modifies copula

## Types of Categorical Proposition

**A** (universal affirmative): All x are y.

**Affirmo:** I affirm

All propositions affirm or deny the predicate of some or all of the subject.

**I** (particular affirmative): Some x are y.

**E** (universal negative): No x are y.

**nEgO:** I deny

**O** (particular negative): Some x are not y.

Note: **Quantity** (universal or particular) affects terms. **Quality** (affirmative or negative) affects copula.

To negate a term, use *non-* or *un-*; to negate the copula, use *no* or *not*.

## Rules of Distribution

Universal subjects (A & E propositions) and negative predicates (E & O propositions) are distributed.

Particular subjects (I and O propositions) and affirmative predicates (A and I propositions) are undistributed.

## Parts of a Syllogism

**(Two Categorical Premises and a Categorical Conclusion)**

Major term: predicate of conclusion; Minor term: subject of conclusion; Middle term: in both premises but not in conclusion

Major premise: major term (P) and middle term (M)

(Note: Middle term links major & minor

Minor premise: minor term (S) and middle term (M)

terms to yield the conclusion.)

Conclusion: minor term (S—because subject of conclusion) and major term (P—because predicate)

## Five Rules of the Syllogism (following Gordon H. Clark)

Breaking the **rule** at left commits the **fallacy** at right:

1. Two negative premises do not imply a conclusion. (G&B 5; Joseph 3)

Exclusive premises

2. Two affirmative premises do not imply a negative conclusion. (G&B 7; J 5)

Negation from affirmation

3. An affirmative and a negative premise do not imply an affirmative conclusion. (G&B 4; J 4)

Affirmation from negation

4. Two premises in both of which the middle term is undistributed do not imply a conclusion. (G&B 2; J 2)

Undistributed middle

5. Two premises in which a given term is undistributed do not imply a conclusion in which it is distributed. (G&B 3; J 6)

Illicit process

## Figures and Their Valid Moods

(64 possible moods in each of 4 figures = **256 possible argument forms**)

(Moods in parentheses yield weaker conclusions than the premises warrant.) (19 strong valid + 5 weak valid = **24 (9.4%) valid forms; 232 [90.6%] invalid**)

	Figure 1	Figure 2	Figure 3	Figure 4
Major Premise	M P	P M	M P	P M
Minor Premise	S M	S M	M S	M S
Valid Moods	Barbara Celarent Darii Ferio (AAI) (EAO)	Cesare Camestres Festino Baroco (EAO) (AEO)	Darapti Disamis Datisi Felapton Bocardo Ferison	Bramantip Camenes Dimaris Fesapo Fresison (AEO)

Vowels denote proposition types (AEIO). Consonants in Figs. 2-4: •Initial consonant matches Fig. 1 target; •"m" (= *muta*): transpose premises; •"s" (= *simpliciter*): convert simply the original premise or new conclusion designated by the preceding vowel; •"p" (= *per accidens*) convert by limitation the original premise or new conclusion designated by the preceding vowel; •"c" (= *conversio syllogismi*): employ indirect reduction (using contradiction of conclusion of original argument as premise in new argument whose conclusion contradicts a premise in the original argument). Moods in parentheses ( ) are "weak" valid, concluding less (the subaltern) than the universal conclusion possible.

## Formal Fallacies of the Categorical Syllogism

**(Fallacies in the Form, Not Substance, of the Argument)**

1. Illicit Major—major term distributed in conclusion but not in premise.

2. Illicit Minor—minor term distributed in conclusion but not in premise.

3. Undistributed Middle—middle term not distributed at least once.

4. Four-term Fallacy—at least one term is ambiguous or equivocal (results in a non-syllogism).

## Immediate Deduction

**(Inferences From a Single Categorical Proposition)**

Obversion: Maintain order, change quality, contradict the predicate. A to E (*All X are Y, ∴ No X are non-Y.*) E to A (*No X are Y, ∴ All X are non-Y.*) I to O (*Some X are Y, ∴ Some X are not non-Y.*) O to I (*Some X are not Y, ∴ Some X are non-Y.*)

Conversion: Reverse subject and predicate, maintain quality, ensure that no term distributed in the converse (result) was undistributed in the convertend (original). A to I (*All X are Y, ∴ Some Y are X.*) E to E (*No X are Y, ∴ No Y are X.*) I to I (*Some X are Y, ∴ Some Y are X.*) **O does not convert.**

Conversion by Negation: Obvert and then convert. A to E (*All X are Y* obverts to *No X are non-Y*, which converts to *No non-Y are X.*) E to I (*No X are Y* obverts to *All X are non-Y*, which converts to *Some non-Y are X.*) O to I (*Some X are not Y* obverts to *Some X are non-Y*, which converts to *Some non-Y are X.*) **I does not convert by negation** because I would obvert to O but O would not convert.

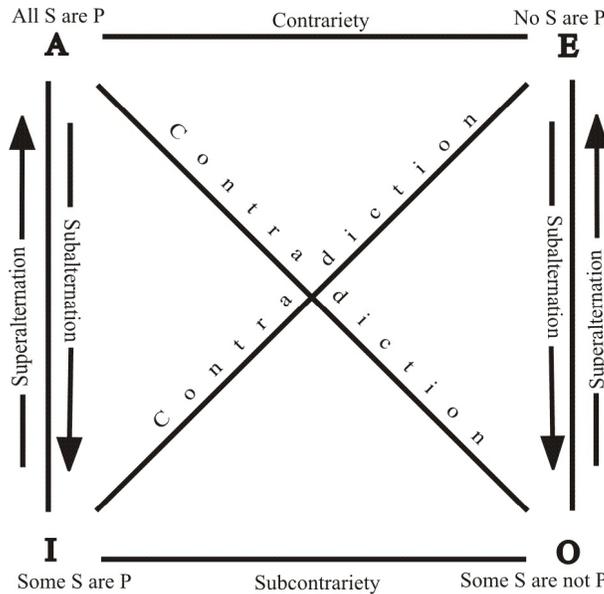
Contraposition: obvert, convert, obvert. A to A (*All X are Y* obverts to *No X are non-Y*, which converts to *No non-Y are X*, which obverts to *All non-Y are non-X*.)  
 E to O (*No X are Y* obverts to *All X are non-Y*, which converts to *Some non-Y are X*, which obverts to *Some non-Y are not non-X*.) O to O (*Some X are not Y*  
 obverts to *Some X are non-Y*, which converts to *Some non-Y are X*, which obverts to *Some non-Y are not non-X*.) **I does not contraposit** because I would obvert to  
 O but O would not convert.

**Seven Relations Between Different Propositions (or Types of Propositions) with the Same Terms**

1. Independence? Truth or falsehood of one is unrelated to truth or falsehood of other: impossible.
2. Equivalence? Both statements *must* be true or both *must* be false, not one true and the other false: impossible.
3. Contradiction: Truth of either statement (A or E; I or O) entails falsehood of other (O or I; E or A), and falsehood of either (A or E; I or O) entails truth of other (O or I; E or A).
4. Contrariety: If either (A or E) is true, other (E or A) must be false, but both (A and E) might be false.
5. Subcontrariety: Truth of either (I or O) does not entail falsity of other (O or I), but falsity of either (I or O) entails truth of other (O or I).
6. Subalternation: Truth of one (A or E) entails truth of other (I or O), but falsity of one (A or E) does not entail falsity of other (I or O).
7. Superalternation: Truth of one (I or O) does not entail truth of other (A or E), but falsity of one (I or O) entails falsity of other (A or E).

**Square of Opposition**

(Illustrates Relations 3-7 above.)



**Other Types of Arguments**

*Hypothetical arguments:* Conditional “major” premise and categorical (unconditional) “minor” premise yield a constructive (affirmative) or destructive (negative) conclusion:

*Modus Ponens* (constructive; way of addition; affirms antecedent):  $p \supset q; p; \therefore q$  (= If p then q; p; therefore q.)

*Modus Tollens* (destructive; way of subtraction; denies consequent):  $p \supset q; \sim q; \therefore \sim p$  (= If p then q; not q; therefore not p.)

Fallacies: denying antecedent ( $p \supset q; \sim p; \therefore \sim q$ ) and affirming consequent ( $p \supset q; q; \therefore p$ ).

*Disjunctive arguments:* Disjunctive “major” premise and categorical “minor” premise yield categorical conclusion.

*Exclusive disjunctives:* either X or Y but not both and not neither; valid: *modus ponendo tollens* (way of denying by affirming): (1) X;  $\therefore \sim Y$ ; (2) Y;  $\therefore \sim X$ ; *modus tollendo ponens* (way of affirming by denying): (1)  $\sim X$ ;  $\therefore Y$ ; (2)  $\sim Y$ ;  $\therefore X$

*Inclusive disjunctives:* either X or Y or both, but not neither; valid: **only modus tollendo ponens**: (1)  $\sim X$ ;  $\therefore Y$ ; (2)  $\sim Y$ ;  $\therefore X$ . Fallacy in *inclusive* disjunctives: inferring, from the affirmation of either alternant, the denial of the other. The only conclusion validly proved in inclusive disjunctives is affirmative.

*Conjunctive argument:* Conjunctive “major” premise and simple “minor” premise yield simple conclusion.

(1)  $\sim(P \ \& \ Q)$ . P.  $\therefore \sim Q$ . (2)  $\sim(P \ \& \ Q)$ ; Q.  $\therefore \sim P$ .

Fallacy: inferring, from denial of either, affirmation of other: (1)  $\sim(P \ \& \ Q)$ ;  $\sim P$ ;  $\therefore Q$ ; (2)  $\sim(P \ \& \ Q)$ ;  $\sim Q$ ;  $\therefore P$ .

*Dilemma:* forcing the opponent to choose among his beliefs.

(1) Constructive: (a) Simple:  $(p \supset q) \ \& \ (r \supset q)$ ;  $p \vee r$ ;  $\therefore q$ ; (b) Complex:  $(p \supset q) \ \& \ (r \supset s)$ ;  $p \vee r$ ;  $\therefore q \vee s$

(v = “or”)

(2) Destructive: (a) Simple:  $(p \supset q) \ \& \ (p \supset r)$ ;  $\sim q \vee \sim r$ ;  $\therefore \sim p$ ; (b) Complex:  $(p \supset q) \ \& \ (r \supset s)$ ;  $\sim q \vee \sim s$ ;  $\therefore \sim p \vee \sim r$

Fallacies: same as in hypothetical arguments: denying the antecedent or affirming the consequent.

Answering dilemmas: (1) Going between the horns: denying the disjunctive premise. (2) Taking the dilemma by the horns: denying the hypothetical premise.

(3) Countering the dilemma: Proposing another dilemma that proves the opposite (contradictory) conclusion from that reached by the first dilemma.

*Sorites:* “a syllogism in the first figure with many middle terms,” i.e., “a polysyllogism in the first figure with the intermediate conclusions suppressed” (H. W. B. Joseph). (Note: sequence does not affect validity.) (Fallacies: same as for categorical syllogism—but with more opportunities to fail.)

Beginning with the minor premise: A is B. B is C. C is D. D is E. E is F. Therefore A is F. Equally valid is any other sequence of the same propositions, e.g.:

A is B. C is D. E is F. B is C. D is E. Therefore A is F.

Beginning with the major premise: E is F. D is E. C is D. B is C. A is B. Therefore A is F. Equally valid is any other sequence of the same propositions, e.g.:

E is F. A is B. D is E. B is C. C is D. Therefore A is F.

*Enthymeme:* Any argument, of any type, in which one or more premises or the conclusion is unexpressed, the arguer depending on the hearer to supply the missing statements.

## Informal Fallacies

[Note: An argument may embody two or more informal fallacies, and the informality leads to some overlapping, so that what might be called, e.g., *ignoratio elenchi* (irrelevant conclusion) might also be called red herring (diverting the issue).]

### A. Fallacies of Relevance

1. *Argumentum ad Baculum*: appeal to force (“Agree with me or I’ll punch you in the nose!”)
2. *Argumentum ad Hominem* (abusive): attacking the person instead of the argument; some subvarieties:
  - a. poisoning the well: direct attack on the trustworthiness of one making an argument
  - b. *Tu quoque!* (“You too!”): “accusing your critic of the same thing your critic accuses you of, rather than defending yourself against the criticism” (Kreeft, 80).
  - c. genetic fallacy: rejecting a person because of his origin or his claim or because of who said it
3. *Argumentum ad Hominem* (circumstantial):
  - a. urging an opponent to accept one’s argument not on its logical validity or evidential credibility but because its conclusion is consistent with other(s) of the opponent’s beliefs, or
  - b. urging an audience to reject an opponent’s argument because its conclusion seems to serve his self-interest
4. *Argumentum ad Ignorantiam* (from ignorance or silence): X must be true because it has not been proved false. Exceptions:
  - a. In court of law, accused is presumed innocent until proven guilty.
  - b. Where it is reasonable to believe that if X were true Y would also be true, the falsehood of Y will (by *modus tollens*) entail reasonable grounds for concluding the falsehood of X.
5. *Argumentum ad misericordiam*: appeal to pity (“Buy my books so I can feed my starving children!”)
6. *Argumentum ad populum*: “the attempt to win popular assent to a conclusion by arousing the feelings and enthusiasms of the multitude” (Copi, *Intro.*, 4<sup>th</sup> ed., 79)
  - a. alternate definition: “deciding truth by opinion polls” (G&B 97)
  - b. related fallacy: *consensus gentium*: appealing to majority opinion (G&B 97)
7. *Argumentum ad verecundiam*: appeal to authority, particularly the famous, without regard to their expertise.
8. *Argumentum ad ignominiam*: appeal to shame instead of reason as the ground for belief
9. *Dicto simpliciter* or *Accident*: “applying a general rule to a particular case whose ‘accidental’ circumstances render the rule inapplicable” (Copi, 81)
10. *Converse accident*: hasty generalization, drawing a general rule from unrepresentative example(s); sometimes called (or committed by) *cliche* or *special case* or *stereotyping*: “makes no room for the exception” (Kreeft, 91)
11. *Black and White*: not allowing for gradations between extremes
12. *False cause*:
  - a. *Non causa pro causa*, mistaking for the cause of an event anything that is not the true cause
  - b. *Post hoc ergo propter hoc*, assuming that because X occurs before Y it must be the cause of Y
13. *Petitio Principii*: begging the question—assumes as a premise the conclusion one intends to prove; related to this, *arguing in a circle* uses the conclusion as a premise for an argument to prove one of the premises for the conclusion
14. *Contradictory Premises*: an argument in which one premise contradicts the other, though normally the contradiction is carefully hidden by obscure language
15. *Complex question*: asking a question that assumes a particular answer to a prior, unspoken question (e.g.: “Have you stopped beating your wife?” and “Where did you hide the murder weapon?”)
16. *Ignoratio Elenchi* (“ignorance of confutation”): irrelevant conclusion; proof of a conclusion not contradictory to the proposition to be confuted. (“Free trade maximizes living standards.” “But foreign tariffs hurt us!”)
17. *Argumentum ab annis*: appeal to age (for or against a conclusion); also called *argumentum ab gerontiam*.
18. *Argumentum ad futuris*: appeal to the future; related: *Argumentum ad novum*: appeal to newness or novelty
19. *Straw man*: setting up a caricature of an opponent’s position and attacking that (cf. B.9, quoting out of context)
20. *Special pleading*: considering only evidence that agrees with one’s conclusion
21. *Red herring*: diverting the issue, drawing attention away from the actual argument

### B. Fallacies of Ambiguity

1. *Equivocation*: using the same word(s) in different senses within one argument “Strict definitions and strict adherence to them are essential to intelligible discussion.”—Gordon H. Clark, *Religion, Reason, and Revelation*, 303.
2. *Amphiboly*: ambiguity not in the meaning of words but in the grammar or syntax of the proposition
3. *Accent*: ambiguity arising from shift in focus or emphasis
4. *Composition* (distinguish from hasty generalization: The latter “argues that since many or most atypical members of a class have a specified property therefore all members of the class (distributively) do so.” The former “that since all of the members of a class have a specified property therefore the class *itself* has that property.” Copi, 99)
  - a. from parts to whole: assuming that what is true of the part(s) is true of the whole
  - b. from individual part(s) to all parts
5. *Division* (distinguish carefully from accident. The latter “argues that since most members of a class have a specified property therefore any particular member or subclass of members . . . must have that property also,” the former “that since a class *itself* (collectively) has a specified property therefore any member or subclass of members of the class must have that property also.” Copi,

99):

- a. assuming that what is true of the whole is true of the part(s)
- b. arguing “from the properties of a collection of elements to the properties of the elements themselves” (e.g., “American Indians are disappearing. Richard is an American Indian. Therefore Richard is disappearing.”)
6. *Slanting*: using commendatory (or condemnatory) language without proving that the thing named is truly commendable (or condemnable)
7. *Slogans*: substituting unexamined, brief expressions (e.g., bumper stickers) for thoughtful argument
8. *Hyperbole*: exaggeration, or “absurd extension” (Distinguish from *reductio ad absurdum*, valid argument proving that if you accept a certain proposition you will be driven by logic to accepting another that is absurd.)
9. *Quoting Out of Context*: implies, by ignoring contextual qualifiers, that an author meant something other than what he really meant. (type of A.19, straw man)
10. *False Assumption*: e.g.: “Does your dog bite?” “Nope.” “Here, pooch, let me scratch your head.” The dog promptly bites. “I thought you said your dog didn’t bite!” “That’s not my dog.”

### C. Reductive Fallacies

1. *Nothing-Buttery*: e.g., man is nothing but matter in motion
2. *Genetic Fallacy*: accept (or reject) this because it comes from that; often *argumentum ad hominem* abusive.
3. *Category Mistake*: an argument depending on the attribution of a predicate (character) suitable to an object of one category to an object of another category; e.g., “If God is all powerful, why can’t He make a rock so big He can’t lift it?” or “. . . why can’t He make a square circle?” (both assuming that the performance of the logically contradictory is an object of power), or “There is only one God, so if the Father is God, the Son can’t be” (assuming that *God* and *person* are the same category of being)
4. *Faulty Analogy*: ignoring relevant differences in analogical reasoning
5. *Argument of the Beard*: “assumes that adding up small differences does not make a big difference. It tells us that if a line is hard to draw, then it is impossible to draw” (G&B 110).

### D. Procedural Fallacies (all from Kreeft, 104ff)

1. Refuting the conclusion instead of the argument
2. Assuming that refuting an argument refutes its conclusion
3. Ignoring the argument (beside the point)
4. Substituting explanation (often analogy) for proofs
5. Answering another argument than the one given
6. Shifting the burden of proof
7. Winning the argument but losing the arguer—or vice versa

### E. Metaphysical Fallacies (all from Kreeft, 109ff)

1. Misplaced concreteness: confusing the abstract with the concrete (“I love humanity. It’s people I hate!”)
2. Accident: confusing the accidental with the essential
3. Confusing quantity with quality (“The embryo can’t be a sacred human being. It’s too tiny.”)
4. Confusing logical, efficient, and psychological causes
5. Confusing essence and existence (existential fallacy)
6. Confusing the natural with the common

### F. Other Types of Fallacies

1. *Faulty Dilemma*: tries to force someone to choose one of two alternatives when (a) there might be more alternatives or (b) it might be possible to embrace both.
2. *Hypothesis Contrary to Fact*: assumes it possible to predict what would have followed had reality been different, when in fact we cannot know. (This is a specialized variety of the fallacy of *denying the antecedent*. It argues, “If what really happened happened, then what really ensued ensued. But suppose what really happened didn’t happen. In that case, what really ensued would not have ensued.”)
3. *Slippery Slope*: argues that one should reject A (of unjudged desirability) because it might lead to B (something undesirable) “when the connection between the two statements is not *logical*, but *psychological* (or sociological, or historical, etc.)” (G&B 113).
4. *Jargon* or *blinding with science*: using technical language to intimidate opponents, impress an audience, or appear to know more than you do. Scholars legitimately use technical terms, but when communicating with non-specialists they should define terms to ensure understanding.
5. *Non sequitur*: an argument fails to establish its conclusion not because it commits a formal fallacy or one of the informal fallacies above but because its propositions bear no logical relation to each other. (E.g.: “Alligators are reptiles. White shirts look nice on men. Therefore democracy is the best form of government.”) Every fallacious inference is a *non sequitur*, but this broad, general name especially fits inferences from logically unrelated premises.

**Sources:** Joseph, H. W. B. *An Introduction to Logic*, 2d ed. (1916) Cresskill, NJ: Paper Tiger, 2000; Geisler, Norman L., and Ronald M. Brooks. *Come, Let Us Reason: An Introduction to Logical Thinking*. Grand Rapids: Baker, 1988; Clark, Gordon H. *Logic*, 2d ed. Jefferson, MD: Trinity Foundation, 1988. Copi, Irving M. *Introduction to Logic*, 4th ed. New York: Macmillan, 1972; Kreeft, Peter. *Socratic Logic*. South Bend: St. Augustine’s Press, 2004.